AP Physics C

Drag Forces

Objects that move through a fluid, gas or liquid, will experience an amount of friction that will increase as the velocity of the object increases. Newton’s second law for this situation when an object falls through a fluid is

**∑ F = ma = mg – FD**

Eventually, as speed increases and **FD** increases, acceleration will decrease and eventually become zero. The velocity at which this occurs is the terminal velocity, ***vT****,*of the object and the object will continue to fall through the fluid at that velocity until it reaches the bottom of the fluid.

The total amount of drag depends upon a couple factors, primarily the cross-sectional area of the object and the viscosity of the fluid. Usually these characteristics are all just rolled into one constant that will be represented within a function that describes **FD.** Drag will usually have a magnitude ***kv*** or ***kv2*** where k would be a constant and ***v*** is the velocity of the object.

Rewriting Newton’s 2nd law yields:

**∑ F = ma = mg – *kv***

This expression can be written as a differential equation that can be used to determine the speed of the box, ***v*** as a function of time ***t.***

***a = dv/dt; acceleration is the derivative of velocity vs time***

**m(*dv/dt*) = mg – *kv***

The expression can easily be used to determine the ***vT***by setting ***dv/dt*** = 0 (a = 0) and solving for ***v***

**0 =** **mg – *kvT***

***kvT =* mg**

***vT =* mg/*k***

This differential equation can also be used to derive an equation for the speed ***v*** of the object as a function of time ***t, or v(t)***.

**m(*dv*) = mg – *kv***

 ***dt***

 ***m (dv) = (dt) (mg – kv)***

 ***(dv) = (dt)***

 ***(mg – kv) m***

**[1/m](dt) = *(dv)* [1/(mg – *kv)]* separate variables**

**1/m =**

] = – [**ln(mg – *kv)***]

* **=** ln() = 1 -

= = 1 -

*v(t) =*  (1 - )

**Problems**

1. A ping pong ball has a mass of 2.3 g and experiences a drag force **FD** expressed in the form of *0.00028v2. D*etermine the terminal velocity of the ping pong ball.
2. A skydiver with a mass of 60.0 kg reaches a terminal velocity of 90 km/hr when she is in a position with her arms and legs outstretched. What is the magnitude of the drag force on the skydiver? If the drag force takes the form of *bv2* what is the value of *b*?
3. A box of mass *m* initially at rest on a horizontal surface is acted upon by a constant force of magnitude *FA* that is parallel to the surface. The friction between the box and the surface is negligible, but the box is subject to a drag force of magnitude *kv* where *v* is the speed of the box and *k* is a positive constant. Express all your answers in terms of the given quantities and fundamental constants, as appropriate.
	1. Write, but do not solve, a differential equation that could be used to determine the speed *v* of the box as a function of time *t.*
	2. Determine the magnitude of the terminal velocity of the box.
	3. Use the differential equation from part (a) to derive the equation for the speed *v* of the box as a function of time *t*. Assume the *v* = 0 at *t* = 0.
	4. On the axes below, sketch a graph of the acceleration, a, of the box and speed *v* of the box as a function of time *t*. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions as appropriate.

a v

t  t