Angular Momentum Conservation Examples

1. A 2.0 kg particle is moving at 4.5 m/s due east along an east-west line. What is the angular momentum of the particle (Magnitude and direction) at a point 6.0 m due north of the particle? What about a point 6.0 m due south of the particle? What about at a point 6.0 m due east of the particle?

Angular momentum of a point particle is: L = *mvr* sin Θ.

6 m due north: 2.0 kg (4.5m/s)6.0 m)(sin 90) = 54 kg m2/s (upward)

6 m due south: 54 kg m2/s (down)

6 m due east: 0 kg m2/s (sin 0 = 0)

1. You stand on a frictionless platform that is rotating at and angular speed of 1.5 rev/s. Your arms are outstretched, and you hold a heavy weight in each hand. The moment of inertia of you, the extended weights and the platform is 6.0 kg∙ m2. When you pull the weights in toward your body, the moment of inertia decreases to 1.8 kg∙m2. What is the resulting angular speed of the platform? What is the change in rotational kinetic energy of the system?

Lo = Lf = Iωo = Iωf 🡪 6.0 kg ∙m2 (1.5 rev/s) /1.8 kg∙ m2 = ωf = 5 rev/s

1. An ice Skater has a mass of 58 kg. (Her body is 50 kg and each arm is 4.0 kg) She has a rotational velocity of 1.5 rev/s when her arms are out stretched. What will her rotational velocity be, in rev/s, when she pulls her arms in next to her body? (Assume her body has a cylindrical radius of 0.20 m and her arms are 1.0 m long).

There is no external torque acting on the skater, so Li = Lf and Iωout =Iωin

Initially, her initial moment of inertia is:

I = Ibody + Iarmsout = ½ mr2body + 2[ 1/3 m L2] (a rod pivoting at one end)

Iarms out = (0.5)(50 kg)(0.2)2 + 2(1/3(4.0)(1)2 =

1.0 kg∙ m2 + 2.67 kg∙ m2 = 3.67 kg∙m2

Iarms in = Ibody + Iarms in = 1.0 kg∙m2 + 2(1/3 (4.0 kg)(0.20)2 = 1.32 kg∙ m2

For the conservation of momentum, Iout ωout = Iin ωin so

ωin = (Iout/Iin)ωout = 3.67/1.32(1.5rev/s) = 4.2 rev/s

1. A particle is attached to a string 2.0 m long and swings in a circle with a velocity of 6.0 m/s. The string is wrapped around a peg and the radius decreases as the mass swings. Find the amount of string that will be wrapped around the peg when the velocity of the mass is 20 m/s.

Io ωo = If ωf Io = mR2 and ωo = v/R = 3 rad/s

Iωf = If = mr22 and ωf = v2/r2

mR2(v/R) = mvR = mr22 (v/r2) = mvr2

r2 = mvR/(mv2) = vR/v2 = 6m/s(2m)/20 m/s = 0.6m so 1.4 m of string is wrapped up.

Two balls, one of mass 1 kg and one of mass 2 kg, are confined to move in a circular track. They move at an equal velocity, *v*, in opposite directions on the track and collide at a point. The two balls stick together. What is the magnitude and direction of the velocity of the balls after the collision, in terms of *v*?

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L2 = mr2ω = 2r2ω = 2r2v/r = 2vr; L1 = mr2ω = 1r2v/r = vr

ΔL = L2 – L1 (since they move in opposite directions

ΔL = 2vr - vr = vr

After they collide: L = mr2ω = 3r2(vf/r) = 3vfr

Setting Lo = Lfvr = 3vfr or v/3= vf

1. A skater spinning at 32.0 rad/s with her arms and leg extended outward. In this position her moment of inertia with respect to the vertical axis about which she is rotating is 45.6 kg∙m2. She pulls her arms and leg in close to her body changing her moment of inertia to 17.5 kg∙m2. What is her new angular velocity?



Ioωo = Ifωf; ωf = (Io/If)ωo = (45.6/17.5)(32.0 rad/s) = 83.4 rad/s



1. A horizontal disc of rotational inertia 4.25 kg∙m2 with respect to its axis of symmetry is spinning counterclockwise about its axis of symmetry, as viewed from above, at 15.5 rev/s on a frictionless massless bearing. A second disc, of rotational inertia 1.80 kg∙m2with respect to its axis of symmetry, spinning clockwise as viewed from above about the same axis (which is also its axis of symmetry) at 14.2 rev/s, is dropped on top of the first disc. The two discs stick together and rotate as one about their common axis of symmetry at what new angular velocity (in rad/s)?

15.5 rev/s (2Π rad/rev) = 97.39 rad/s

14.2 rev/s(2Π rad/rev) = 89.22 rad/s

*Lo* = *Lf*

I1ω1 + I2ω2 = (I1 + I2)ω’

4.25 kg∙m2( – 97.39 rad/s) + 1.80 kg∙m2(89.22 rad/s) = (6.05 kg∙m2) ω’

(– 413.9 + 160.0)/6.05 kg∙m2=ω’= – 41.9 rad/s (CCW)

1. NASA planned to put a satellite into a circular orbit around Pluto, but something went awry and the satellite ended up in an eliptical orbit around the dwarf planet. At its nearest point to the planet, 6 x 106 m the satellite has a velocity of 9000 m/s. find the vleocity of the satellite at its farthest point from Pluto, 2 x 107 m.

I1ω1 = I2ω2 ; mv1r1 = mv2r2 so

v2 = v1(r1/r2) = 9000 (6 x 106/2 x 107) = 2700 m/s